Generation of electromagnetic fields in string cosmology with a massive scalar field on the anti D-brane

Mohammad R. Garousi[1], M. Sami[2] and Shinji Tsujikawa[3]
[1] *Department of Physics, Ferdowsi University, P.O.Box 1436, Mashhad, Iran*
and
Institute for Studies in Theoretical Physics and Mathematics IPM, P.O.Box 19395-5531, Tehran, Iran
[2] *IUCAA, Post Bag 4, Ganeshkhind, Pune 411 007, India*
[3] *Department of Physics, Gunma National College of Technology, 580 Toriba, Maebashi, Gunma 371-8530, Japan*

(February 1, 2008)

We study the generation of electromagnetic fields in a string-inspired scenario associated with a rolling massive scalar field ϕ on the anti-D3 branes of KKLT de Sitter vacua. The 4-dimensional DBI type effective action naturally gives rise to the coupling between the gauge fields and the inflaton ϕ, which leads to the production of cosmological magnetic fields during inflation due to the breaking of conformal invariance. We find that the amplitude of magnetic fields at decoupling epoch can be larger than the limiting seed value required for the galactic dynamo. We also discuss the mechanism of reheating in our scenario and show that gauge fields are sufficiently enhanced for the modes deep inside the Hubble radius with an energy density greater than that of the inflaton.

Introduction – The development of string theory has continuously stimulated its application to cosmology [1]– especially to cosmic inflation. Given the fact that the detection of some stringy effects in accelerators is difficult in the foreseeable future, string cosmology presumably provides the only way to test the viability of string/M-theory concretely. Fortunately the recent measurement of the Cosmic Microwave Background (CMB) [2] has brought the high-precision cosmological dataset by which inflationary models can be seriously constrained [3].

The development in compactification with flux and branes has provided examples of dynamics that fix all moduli [4,5]. In particular, the authors of Ref. [5] showed that by adding anti-D3 branes to the solutions of [4], one can lift the supersymmetric vacuum energy and find locally stable minima with positive cosmological constant (KKLT vacua).

Lately we proposed a string-inspired cosmological model [6] based on a massive Dirac-Born-Infeld (DBI) [7] scalar field ϕ rolling on the anti-D3 brane of KKLT vacua (see also Ref. [8]). We showed that this scenario satisfies observational constraints coming from the CMB temperature anisotropies by evaluating the spectra of scalar and tensor perturbations generated during the rolling scalar inflation [6]. The problem of reheating associated with the DBI action [9] is overcome by taking into account a negative cosmological constant of the KKLT vacua. The origin of dark energy can be explained if the potential energy does not exactly cancel with the negative cosmological constant at the potential minimum.

In this work we shall consider several important cosmological aspects which distinguish our scenario from others. It is well known that electromagnetic fields are not generated in a Friedmann-Robertson-Walker (FRW) background if the underlying theory is conformally invariant as in the classical electrodynamics. On the other hand we observe magnetic fields with magnitudes of order 10^{-6} G on scales larger than 10 kpc [10]. A number of people tried to solve this discrepancy by breaking conformal invariance of the theory [11] or by breaking conformal flatness of the background geometry [12,13]. In spite of these attempts it is fair to say that it is not easy to construct a satisfactory cosmological model based on particle physics which explains the origin of seed magnetic fields and also satisfies other observational constraints. Presumably a unique example known earlier, is provided by Gasperini *et al.* [14] who highlighted the coupling between dilaton and gauge fields based upon the Pre-Big-Bang scenario. This model respects conformal invariance which allows the photon to decouple from the metric.

The DBI action in our string-inspired scenario involves the coupling between gauge fields and the inflaton field ϕ. We show that this coupling naturally leads to the production of cosmological magnetic fields during inflation and the amplitude can be greater than the limiting value of the seed fields for galactic dynamo. We also study the reheating dynamics in which radiation is generated very efficiently through the amplification of gauge fields.

Model – Let us consider a massive open string excitation of the anti-D3 brane of KKLT vacua as a candidate of the inflaton ϕ. In addition to the massive inflaton potential $V(\phi)$, we implement a negative cosmological constant $(-\Lambda)$ which comes from the stabilization of the modulus fields [4,5]. Then the DBI type effective 4-dimensional action for our system is described by [6]

$$\mathcal{S} = \int d^4x \bigg\{ \sqrt{-g}\left(M_p^2 R/2 + \Lambda\right) \\ -V(\phi)\sqrt{-\det(g_{\mu\nu}+\partial_\mu\phi\partial_\nu\phi+F_{\mu\nu})} \bigg\}, \qquad (1)$$

where $V(\phi) = \beta^2 T_3 e^{\frac{1}{2}m^2\beta\phi^2}$, M_p is the reduced Planck

mass, and $F_{\mu\nu} \equiv 2\nabla_{[\mu}A_{\nu]}$ is the Maxwell tensor with A_μ the four-potential. Here T_3 is a brane tension, β is a warp factor, and m is the mass of scalar vertex operators [6].

In a spatially flat FRW metric with a scale factor a, we get the background equations of motion

$$H^2 = \frac{1}{3M_p^2}\left[V(\phi)/\sqrt{1-\dot{\phi}^2} - \Lambda\right], \tag{2}$$

$$\ddot{\phi}/(1-\dot{\phi}^2) + 3H\dot{\phi} + \beta m^2\phi = 0\,, \tag{3}$$

where $H \equiv \dot{a}/a$ is the Hubble rate and a dot denotes the derivative in terms of cosmic time t. Here we dropped the contribution from gauge fields. During inflation the negative cosmological constant is negligible relative to $V(\phi)$ and inflation is realized for $\dot{\phi}^2 < 2/3$. The presence of a negative cosmological constant leads to a successful reheating in which the energy density of ϕ scales as a pressureless dust. In what follows we consider the generation of electromagnetic fields during inflation and reheating.

Cosmological magnetic fields generated in inflation – The action to the second order of gauge field becomes

$$\mathcal{S}_g = \int d^4x \frac{V(\phi)a}{2}\sqrt{1-\dot{\phi}^2}\left(\frac{\dot{A}^2}{1-\dot{\phi}^2} - \frac{\vec{\nabla}A\cdot\vec{\nabla}A}{a^2}\right), \tag{4}$$

where we used the coulomb gauge ($A_0 = 0 = \vec{\nabla}\cdot\vec{A}$), and $\vec{A} \equiv A$. From this Lagrangian we get the equation for the each Fourier component of the gauge field

$$\ddot{A}_k + H(1-3\dot{\phi}^2)\dot{A}_k + (1-\dot{\phi}^2)(k^2/a^2)A_k = 0\,, \tag{5}$$

where k is a comoving wavenumber. Introducing a new quantity $\tilde{A}_k = b^{1/2}A_k$ with $b = V(\phi)/\sqrt{1-\dot{\phi}^2}$, we find

$$\tilde{A}_k'' + \left[k^2 - f(\eta)\right]\tilde{A}_k = 0\,, \tag{6}$$

where a prime denotes the derivative with respect to a conformal time $\eta = \int a^{-1}\mathrm{d}t$ and

$$f(\eta) = k^2\dot{\phi}^2 - 3a^2\dot{\phi}\left(H\ddot{\phi} + \frac{H^2\dot{\phi}}{2} + \frac{\dot{H}\dot{\phi}}{2} - \frac{3H^2\dot{\phi}^3}{4}\right). \tag{7}$$

In an asymptotic past ($\eta \to -\infty$) with $\dot{\phi} \to 0$, $f(\eta)$ is vanishingly small compared to k^2 and the solution is given by $\tilde{A}_k^i = e^{-ik\eta}/\sqrt{2k}$. The $\dot{\phi}$ term gradually becomes important during inflation, which leads to the variation of $\tilde{A}_k$. The equation (6) has the following solution:

$$\tilde{A}_k = \alpha_k\tilde{A}_k^i + \beta_k\tilde{A}_k^{i*}\,, \tag{8}$$

where the Bogolyubov coefficient β_k is

$$\beta_k = -i\int_{\eta_i}^{\eta}\tilde{A}_k^i f(\eta')\tilde{A}_k\mathrm{d}\eta'\,. \tag{9}$$

The spectrum of the gauge field A_k can be evaluated analytically by using the method in Ref. [15]. After the Hubble radius crossing ($k \lesssim aH$), the mass term in Eq. (6) is approximately given by $-f(\eta) \simeq -k^2\dot{\phi}^2 + (3/2)\dot{\phi}^2(aH)^2 \sim \dot{\phi}^2/\eta^2$ under a slow-roll approximation. We shall consider a situation in which the field ϕ rapidly decays to radiation after inflation and the gauge field becomes effectively massless during the subsequent radiation dominant era. Then one can find the spectrum of super-Hubble gauge fields by matching two solutions between inflation and radiation (see sec. 4.2 of [15]). This gives the spectrum $\mathcal{P}_{\tilde{A}_k} \equiv k^3/(2\pi^2)|\tilde{A}_k^2| \propto k^n$ with n of order $\dot{\phi}^2$ [see Eq. (90) of [15]]. Since $\dot{\phi}^2$ is much smaller than 1 (see Fig. 1), the resulting spectrum is nearly scale-invariant. In the case of Z-boson with mass M_Z the spectral index is $n = 2(M_Z/H)^2$ [15]. There exists an interesting mechanism to amplify magnetic fields by parametric resonance during reheating using a coupling between a gauge field and a Klein-Gordon scalar [16]. In this case the effective mass of gauge fields during inflation is typically required to be larger than H for parametric resonance to be efficient, thereby giving a blue-spectrum $n > 1$.

The energy density stored in a magnetic field mode $|B_k|$ is given by [13]

$$\rho_B = |B_k|^2/(8\pi) = \omega^4|\beta_k|^2\,, \tag{10}$$

where $\omega = k/a$. In the presence of the $f(\eta)$ term, $|\beta_k|^2$ is not generally zero, which corresponds the generation of electromagnetic fields. Making use of Eqs. (9) and (10) the amplitude of the magnetic field is expressed as

$$|B_k| = \sqrt{8\pi}\frac{k^2}{a^2}|\beta_k| = \sqrt{2\pi}\frac{k}{a^2}\left|\int_{\eta_i}^{\eta_f} Q_k^i f(\eta)Q_k\mathrm{d}\eta\right|\,, \tag{11}$$

where we introduced a dimensionless quantity $Q_k = \sqrt{2k}\tilde{A}_k$. Note that η_f is the time after which the particle creation ceases. We are interested in the amplitude $|B_k^{dec}|$ on a scale corresponding to the time at decoupling, i.e., $\omega_{dec} = k_{dec}/a_{dec} \simeq 10^{-33}$ GeV. From the above equation we obtain

$$|B_k^{dec}| = \sqrt{2\pi}\omega_{dec}H_e(a_e/a_{dec})\alpha\,, \tag{12}$$

where α is a dimensionless quantity, given by

$$\alpha = \left|\int_{\eta_i}^{\eta_f} Q_k^i g(\eta)Q_k\frac{(aH)^2}{a_eH_e}\mathrm{d}\eta\right|\,. \tag{13}$$

Here $g(\eta) = (aH)^{-2}f(\eta)$ and the subscript 'e' denotes the value at the end of inflation.

Let us estimate the amplitude of cosmological magnetic fields generated in our scenario. The slow-roll parameter $\epsilon = -\dot{H}/H^2$ becomes greater than 1 when $\dot{\phi}^2$ reaches $\dot{\phi}^2 \simeq 2/3$, from which one can find the value of ϕ at the end of inflation as $\phi_e = 6.715/(\sqrt{\beta}m)$ by using Eqs. (2) and (3). The COBE normalization gives $\beta \simeq 10^{-9}$ for an exponential potential [6], thereby yielding $H_e \simeq 6.0\times10^{-5}M_p \simeq 10^{14}$ GeV.

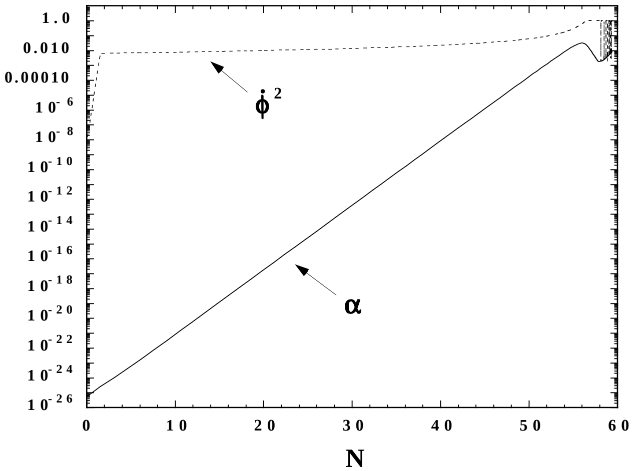

FIG. 1. The evolution of α and $\dot{\phi}^2$ during inflation for $\beta = 10^{-9}$ as a function of the e-folds $N = \int H dt$. We choose the cosmological mode which crossed the Hubble radius about 56 e-folds before the end of inflation. We find that α reaches of order 0.01 at the end of inflation.

The ratio a_e/a_{dec} depends on the details of reheating. For example if the energy density of the field ϕ at the end of inflation is converted to almost instantaneously to radiation, $\rho_R = (\pi^2/30)g_* T_R^4$, where $g_* \sim 100$ is the number of relativistic degree of freedom, then we obtain the reheating temperature $T_R \sim 10^{15}$ GeV for $\beta = 10^{-9}$, which yields $a_e/a_{dec} \sim T_{dec}/T_R \sim 10^{-25}$. When the thermalization process is delayed, one gets the smaller reheating temperature, thereby giving larger values of T_{dec}/T_R. By Eq. (12) the amplitude of the cosmological magnetic field at decoupling epoch is estimated as

$$\left|B_k^{dec}\right| \simeq (a_e/a_{dec})\alpha \ \ \mathrm{G}\,, \tag{14}$$

where we used $1\,\mathrm{G}^2/8\pi = 1.9089 \times 10^{-40}\,\mathrm{GeV}^4$.

The size of α acquired during inflation is crucially important to estimate the amplitude of magnetic fields. As shown in Fig. 1, α rapidly grows during the inflationary stage. This comes from the fact that the second term in Eq. (7) increases due to the presence of the a^2 term, which becomes dominant after the Hubble radius crossing even if the $\dot{\phi}^2$ term is suppressed during inflation. We choose the initial condition $\dot{\phi}_i = 0$ in our numerical simulations, but $\dot{\phi}^2$ is of order 0.01 in most stage of inflation as seen in Fig. 1. The evolution of α becomes mild during the reheating phase and the particle creation ends when the system approaches another asymptotic limit $\eta_f \to \infty$ characterized by $\dot{\phi} \to 0$.

In the case of instant reheating with $\beta = 10^{-9}$, one has $a_e/a_{dec} \sim 10^{-25}$, yielding $|B_k^{dec}| \sim 10^{-27}$ G for $\alpha = 0.01$. It is known that seed fields with the amplitude larger than $|B_k^{dec}| = 10^{-23}$ G is required for the galactic dynamo for a flat universe without a cosmological constant, but this limit is relaxed up to $|B_k^{dec}| = 10^{-30}$ G in the presence of a cosmological constant with $\Omega_\Lambda = 0.7$ [17]. Therefore the value $|B_k^{dec}| \sim 10^{-27}$ G is greater than this limiting value. Thus our scenario provides a sound mechanism to generate seeds magnetic fields through the breaking of conformal invariance. Larger magnetic fields can be obtained if the ratio a_e/a_{dec} is higher and this depends on the details of reheating.

<u>Reheating from electromagnetic fields</u> – Let us next consider how the gauge fields are produced in reheating stage after inflation. We denote the comoving wavenumber corresponding to the Hubble radius at the end of inflation as $k = a_e H_e$. Then the energy density of produced gauge fields can be expressed as

$$\rho_B(k) = \left(\frac{k}{a_e H_e}\right)^4 \left(\frac{a_e}{a}\right)^4 |\beta_k|^2 H_e^4\,. \tag{15}$$

One can estimate the size of gauge fields generated during *inflation* for the mode $k \sim a_e H_e$. Since the frequency of $\tilde{A}_k$ is approximately given by $\omega_k^2 \simeq k^2(1-\dot{\phi}^2)$ for these modes during inflation, the Bogolyubov coefficient β_k at the end of inflation is estimated as

$$\beta_k \simeq (1/4)\langle\dot{\phi}^2\rangle \left(e^{-2ik\eta_e} - e^{-2ik\eta_i}\right)\,, \tag{16}$$

where we used $\tilde{A}_k \simeq \tilde{A}_k^i$. Note that $\langle\dot{\phi}^2\rangle$ is the average value of $\dot{\phi}^2$, which means that $|\beta_k|^2 \ll 1$ at the end of inflation for the mode $k \sim a_e H_e$. Therefore $\rho_B(k)$ is much smaller than the energy density of the field ϕ (ρ_ϕ) at the beginning of reheating.

When the reheating stage starts, the $\dot{\phi}^2$ term rapidly grows to unity as seen in Fig. 1. We numerically found that $\dot{\phi}$ oscillates between -1 and 1 with a slow adiabatic damping due to cosmic expansion. During the oscillation, $\dot{\phi}$ tends to stay around $\dot{\phi} \simeq \pm 1$ for some moment of time as is checked from Eq. (3). While this oscillation is not sinusoidal, one may expect that gauge fields are generated by parametric resonance due to the non-adiabatic change of the frequency in $\tilde{A}_k$.

Actually this happens depending on the size of the momentum k. The gauge fields $\tilde{A}_k$ for the modes deep inside the Hubble radius ($k \gg a_e H_e$) exhibit rapid growth, whereas the modes corresponding to $k \lesssim a_e H_e$ are not amplified. This can be understood as follows. Introducing a new quantity, $\bar{A}_k = a^{1/2}\tilde{A}_k$, the equation for gauge fields for the mode $k \gg a_e H_e$ is approximately given as

$$\ddot{\bar{A}}_k + (k^2/a^2)(1-\dot{\phi}^2)\bar{A}_k \simeq 0\,, \tag{17}$$

which has an oscillating term $\dot{\phi}^2$. The approximate time scale for the oscillation of the field ϕ is $m_{\rm eff}^{-1}$ with $m_{\rm eff} \equiv \sqrt{\beta}m$. We shall define the effective resonance parameter

$$q \equiv \frac{k^2}{4a^2 m_{\rm eff}^2} = \frac{1}{4}\left(\frac{k}{a_e H_e}\right)^2 \left(\frac{a_e}{a}\right)^2 \left(\frac{H_e}{m_{\rm eff}}\right)^2\,, \tag{18}$$

which is analogous to the one defined in Ref. [18].

The parameter q characterises the strength of parametric resonance. The modes deep inside the Hubble radius ($k \gg a_e H_e$) correspond to the large resonance parameter ($q \gg 1$) at the of inflation (here we used the fact that

H_e is the same order as $m_{\rm eff}$). As shown in Fig. 2 we find that the production of gauge fields for the modes $k \gtrsim 10 a_e H_e$ is so efficient that its energy density can surpass that of the field ϕ. Note that there is no amplification of the fluctuation $\delta\phi$ in our scenario at the linear level [6]. The excitation of gauge fields continues until the coherent oscillation of the field ϕ is broken by the backreaction effect of produced particles or the parameter q drops down to of order unity. For the modes $k \lesssim a_e H_e$ the resonance parameter q is smaller than unity, which means that the particle creation is not efficient as seen in Fig. 2. The cosmological magnetic field is involved in this case ($k \ll a_e H_e$), implying that its generation is weak during reheating.

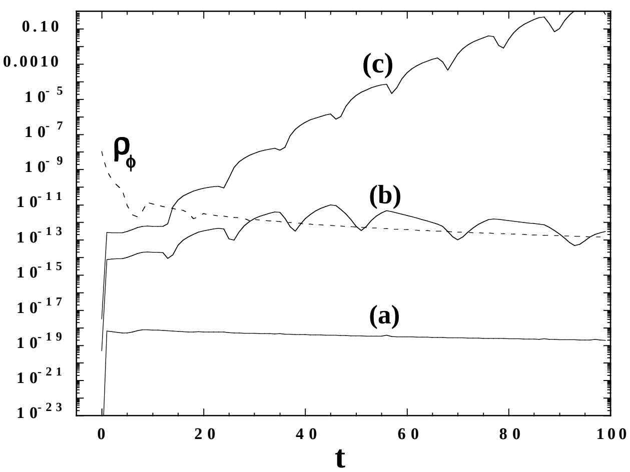

FIG. 2. The evolution of $\rho_B(k)$ during reheating with $\beta = 10^{-9}$ for (a) $k/(a_e H_e) = 1$, (b) $k/(a_e H_e) = 10$ and (c) $k/(a_e H_e) = 20$. We also show the evolution of the energy density for the field ϕ (denoted by ρ_ϕ). We find that $\rho_B(k)$ surpasses ρ_ϕ for the mode $k/(a_e H_e) > 10$.

In Fig 2 we do not account for backreaction and rescattering effects of created particles. This is particularly important when $\rho_B(k)$ catches up ρ_ϕ. In addition the effect of conductivity is not negligible if charged particles are produced after inflation. This typically suppresses the growth of gauge fields [13], whose effect can be important after the intermediate stage of reheating. While the explosive production of gauge fields at the initial stage is robust, we require a fully nonlinear analysis including the effect of conductivity in order to understand the reheating dynamics completely. The process of the thermalization is also dependent on the details of such a nonlinear stage. It is of interest to do a detailed analysis in such a regime, since the reheating temperature is relevant to estimate the amplitude of cosmological magnetic fields.

Conclusions – Generally it is difficult to construct a viable cosmological model in string theory which satisfies all cosmological/observational constraints including inflation, reheating, dark energy and primordial magnetic fields. However our scenario using a massive rolling scalar on the anti-D3 brane of KKLT vacua can provide a satisfactory explanation for the above requirements.

The crucial point for the generation of cosmological magnetic fields in our model is that the DBI type action breaks conformal invariance due to the coupling between gauge fields and the inflaton. This also happens for the rolling tachyon field [19], implying that magnetic fields can be generated during tachyon inflation. Nevertheless tachyon inflation generally suffers from the problems associated with large density perturbations and reheating [9]. In our scenario these problems are overcome by considering a warp metric with a small parameter $\beta (\ll 1)$ and also by a negative cosmological constant, both appearing in the KKLT vacua [5].

During reheating we find that radiation is efficiently produced by the amplification of gauge fields on sub-Hubble scales. This corresponds to the "preheating" stage as in the bosonic particle creation through the coupling $(1/2) g^2 \phi^2 \chi^2$ [18]. The important point is that we used only the DBI type string effective action without adding any phenomenological terms. It is of interest to extend our analysis to the production of fermions using the couplings appearing in string theory.

[1] See e.g., J. E. Lidsey, *et al.*, Phys. Rept. **337**, 343 (2000); M. Gasperini and G. Veneziano, Phys. Rept. **373**, 1 (2003).
[2] D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).
[3] H. V. Peiris *et al.*, Astrophys. J. Suppl. **148**, 213 (2003).
[4] S. B. Giddings *et al.*, Phys. Rev. D **66**, 106006 (2002).
[5] S. Kachru, *et al.*, Phys. Rev. D **68**, 046005 (2003).
[6] M. R. Garousi *et al.*, hep-th/0402075.
[7] M. R. Garousi, JHEP **0312**, 036 (2003).
[8] A. Sen, JHEP **0210**, 003 (2002).
[9] L. Kofman and A. Linde, JHEP **0207**, 004 (2002); M. Sami *et al.*, Phys. Rev. D **66**, 043530 (2002).
[10] Y. Sofue *et al.*, Ann. Rev. Astron. Astrophys. **24**, 459 (1986).
[11] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988); B. Ratra, Astrophys. J. **391**, L1 (1992); A. Dolgov, Phys. Rev. D **48**, 2499 (1993); K. Enqvist *et al.*, hep-ph/0404269.
[12] A. L. Maroto, Phys. Rev. D **64**, 083006 (2001).
[13] B. A. Bassett *et al.*, Phys. Rev. D **63**, 103515 (2001).
[14] M. Gasperini *et al.*, Phys. Rev. Lett. **75**, 3796 (1995).
[15] K. Dimopoulos *et al.*, Phys. Rev. D **65** (2002) 063505; A. C. Davis *et al.*, Phys. Lett. B **501** (2001) 165.
[16] F. Finelli and A. Gruppuso, Phys. Lett. B **502**, 216 (2001).
[17] A. C. Davis *et al.*, Phys. Rev. D **60**, 021301 (1999).
[18] L. Kofman *et al.*, Phys. Rev. D **56**, 3258 (1997).
[19] A. Sen, JHEP **0204**, 048 (2002); JHEP **0207**, 065 (2002).